

Prove that $\frac{1}{1 \times 5} + \frac{1}{5 \times 9} + \frac{1}{9 \times 13} + \dots + \frac{1}{(4n-3) \times (4n+1)} = \frac{n}{4n+1}$ for all integers $n \geq 1$

SCORE: ____ / 10 PTS

by mathematical induction, showing all steps demonstrated in lecture.

BASIS STEP:

If $n = 1$, $\frac{1}{1 \times 5} = \frac{1}{5}$ $\frac{1}{4(1)+1} = \frac{1}{5}$ (1)

INDUCTIVE STEP:

(1) Assume $\frac{1}{1 \times 5} + \frac{1}{5 \times 9} + \frac{1}{9 \times 13} + \dots + \frac{1}{(4k-3) \times (4k+1)} = \frac{k}{4k+1}$ for some arbitrary but particular integer k (1)

(2) $\frac{1}{1 \times 5} + \frac{1}{5 \times 9} + \frac{1}{9 \times 13} + \dots + \frac{1}{(4k-3) \times (4k+1)} + \frac{1}{(4k+1) \times (4k+5)}$

(1) $= \frac{k}{4k+1} + \frac{1}{(4k+1)(4k+5)}$

FIRST & LAST 2 TERMS
MUST BOTH BE SHOWN

$= \frac{k(4k+5)+1}{(4k+1)(4k+5)}$

(1) $= \frac{4k^2 + 5k + 1}{(4k+1)(4k+5)}$

(1) $= \frac{(4k+1)(k+1)}{(4k+1)(4k+5)}$

$= \frac{k+1}{4k+5}$

(1) $= \frac{k+1}{4(k+1)+1}$

(1) By mathematical induction, $\frac{1}{1 \times 5} + \frac{1}{5 \times 9} + \frac{1}{9 \times 13} + \dots + \frac{1}{(4n-3) \times (4n+1)} = \frac{n}{4n+1}$ for all integers $n \geq 1$

Evaluate $\sum_{k=1}^{200} (8k - 3k^2)$. Your final answer must be a number (not involving arithmetic operations).

SCORE: ____ / 4 PTS

$$= 8 \sum_{k=1}^{200} k - 3 \sum_{k=1}^{200} k^2 \quad \textcircled{1}$$

$$= 8 \cdot \frac{1}{2} (200)(201) - 3 \cdot \frac{1}{6} (200)(201)(401) \quad \textcircled{1}$$

$$= -7899300 \quad \textcircled{1}$$

If $f(x) = x^4$, expand and completely simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$.

SCORE: ____ / 4 PTS

$$\frac{(x+h)^4 - x^4}{h} = \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= 4x^3 + 6x^2h + 4xh^2 + h^3$$

Expand and completely simplify the complex number $(3 - 2i)^5$.

SCORE: ____ / 7 PTS

$$1(3)^5(-2i)^0 + 5(3)^4(-2i)^1 + 10(3)^3(-2i)^2 + 10(3)^2(-2i)^3 + 5(3)^1(-2i)^4 + 1(3)^0(-2i)^5$$

$$= 243 + 5(81)(-2i) + 10(27)(-4) + 10(9)(8i) + 5(3)(16) - 32i$$

$$= 243 - 810i - 1080 + 720i + 240 - 32i$$

$$= -597 - 122i$$

Find the 7th term of $(8b - 11g)^{25}$. Your final coefficient may be in factored form as shown in lecture.

SCORE: ____ / 5 PTS

$${}_{25}C_6 (8b)^{25-6} (-11g)^6$$

$$= \frac{25!}{6!19!} (8b)^{19} (-11g)^6$$

PLUS $\frac{1}{2}$ IF YOUR FINAL ANSWER IS POSITIVE

$$= \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19!}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 19!} 8^{19} 11^6 b^{19} g^6$$

$$= 25 \cdot 23 \cdot 22 \cdot 7 \cdot 2 \cdot 8^{19} \cdot 11^6 b^{19} g^6 = 177100 \cdot 8^{19} \cdot 11^6 b^{19} g^6$$